

## Force analysis and simulation – experimental research on the measurement of cylindrical surface profiles

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### Abstract

The results of tests presented herein can have practical applications for the adjustment of rotary speed to ensure constant contact between the measuring sensor's spindle tip and the crankshaft journal of a piston energy converter, whose roundness profile is being measured. Analytical considerations have been supported by the results of simulations as well as experimental tests. The research has also shown that an increase in rotary speed affects the obtained profile shape and the value of determined roundness deviation.

### Introduction

Profiles of cylindrical surfaces are measured by either non-reference or reference methods. During such measurements, a tested object rotates and the sensor spindle makes a relative movement, or the spindle simultaneously moves axially and around the stationary object being measured. In modern measuring instruments or setups, the rotary movement of the object or sensor relative to each other takes place automatically.

To what extent the object's physical profile is reproduced depends on the distribution of forces resulting from the interaction between the measured object and the measuring sensor spindle. A loss of contact results in the zeroing of mutual forces between the measuring spindle and measured object interaction at their contact point (creating the assumption of a lack of surface deformations of the interacting elements). In practice such issues are not analyzed. It is obligatorily assumed that interaction between the spindle and measured object cannot occur. However, research shows that such a possibility does exist, especially when the measured profile is characterized by substantial irregularity and

significant values of geometrical deviations (Fita, 1977; Żebrowska-Łucyk, 1997).

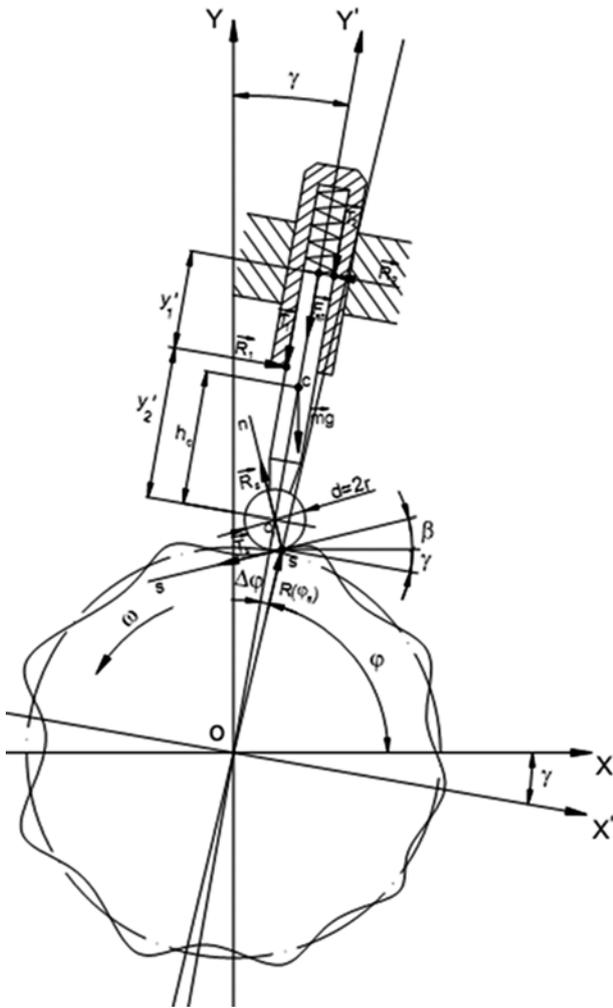
If we consider the mutual interaction between the sensor spindle and the measured object, we may arrive at the conclusion that their physical contact may be interrupted when:

- rotary speed is too high, which may cause the spindle to 'leap' over an irregularity, thus essential details of the profile will not be recorded;
- rotary speed is so high that the inertial mass force of the spindle will be higher than the pressure exerted on the measuring spindle.

Both of these reasons for which the spindle may lose contact with the measured object are directly linked to the rotary speed of relative motion, measuring pressure, diameter, and profile irregularity of the object (Whitehouse, 1990; 1996; Pawlus & Śmieszek, 2005; Tian et al., 2009).

### Analysis of forces in the measurement system

For an analysis of forces occurring at the interaction of the measured object-sensor spindle we can use the diagram shown in Figure 1.



**Figure 1. Distribution of forces at the measured object-spindle interaction**

The diagram provides a basis for the formulation of the following relations between the forces, resulting from the kinetostatic equilibrium of the measuring tip:

$$R_1 - R_2 + mg \sin \gamma - R_s \sin(\beta + \gamma) - T_s \cos(\beta + \gamma) = 0 \quad (1)$$

$$-T_1 - T_2 - F_{sp} - mg \cos \gamma + R_s \cos(\beta + \gamma) - T_s \sin(\beta + \gamma) - ma_{O1} = 0 \quad (2)$$

$$-T_s r - mgh_c \sin \gamma - R_1 y_2' + (T_1 + T_2) \frac{d_1}{2} + R_2 (y_1' + y_2') = 0 \quad (3)$$

where friction forces:  $T_1 = R_1 \mu$ ,  $T_2 = R_2 \mu$ ,  $T_s = R_s \mu$ .

The condition under which the measuring tip will lose contact with the measured profile is when the normal reaction  $R_s$  is equal to zero, hence the equations describing such a state can be written as follows:

$$R_1 - R_2 + mg \sin \gamma = 0 \quad (4)$$

$$-R_1 \mu - R_2 \mu - k(OO_1 - OO_{10}) - mg \cos \gamma - ma_{O1} = 0 \quad (5)$$

$$R_1 \left( \mu \frac{d_1}{2} - y_2' \right) + R_2 \left( y_1' + y_2' - \mu \frac{d_1}{2} \right) - mgh_c \sin \gamma = 0 \quad (6)$$

After substitutions and transformations, we can determine the value of acceleration,  $a_{O1}$ , of the center of the sensor's measuring tip resulting from its relative motion along the curvature of the profile:

$$a_{O1} = (\sin \gamma \mu - \cos \gamma) + \frac{2g \sin \gamma \mu}{y_1'} \left( \frac{\mu d_1}{2} - y_2' + h_c \right) - \frac{k}{m} (OO_1 - OO_{10}) \quad (7)$$

According to the commonly accepted theory of harmonic analysis of roundness profiles, any measured roundness profile  $R(\varphi)$  can be written as this relation (Adamczak, Domagalski & Janecki, 1988; Adamczak, 1988; 2008; Nozdrzykowski, 2013):

$$R(\varphi) = R_o + \sum_{n=2}^k C_n \cdot \cos n(\varphi - \varphi_n) \quad (8)$$

where:

$R_o$  – radius of mean circle;

$C_n$  – amplitude of harmonic component  $n$  of the profile;

$\varphi_n$  – phase shift of the harmonic component  $n$ ;

$\varphi$  – instantaneous angle of rotation;

$n$  – number of harmonic component.

An instantaneous change of the sensor spindle displacement value (path of sensor displacements) depends on the measured profile, radius,  $r$ , of the measuring tip, and angle,  $\gamma$ , defining the direction of spindle displacements in the adopted coordinate system.

If we assume a constant value of the radius,  $R_o$ , an instantaneous change of the spindle displacement value may correspond to a change in distance,  $OO_1$ . That distance, with the relations resulting from the diagram in Figure 1, may be described by the following function:

$$OO_1 = R(\varphi) \sqrt{1 - \frac{r^2 \tan^2(\beta + \gamma)}{R^2(\varphi) (1 + \tan^2(\beta + \gamma))}} + \frac{r}{\sqrt{1 + \tan^2(\beta + \gamma)}} \quad (9)$$

Relation (9) determines the path of sensor spindle displacements expressed by the means of parameters

describing the measured profile, parameters of the measuring system (radius,  $r$ , and angle,  $\gamma$ ), and an instantaneous value of angle,  $\varphi$ , of the measured profile presented in the polar coordinate system.

Making a double differentiation of function (9) in regards to  $\varphi$  and assuming that at a constant angular speed,  $\omega$ , the quotient  $d\omega/d\varphi = 0$ , we get a relation determining the value of acceleration,  $a_{o1}$ , corresponding to the acceleration determined from the previous relation (7).

Comparing relations (7) and double differentiation (9) and assuming that  $\gamma = 0$ ,  $r = \text{const.}$ , we obtain the following functional relation:

$$n_o = f(R_o, C_n, n, \varphi_n, \varphi, P_k) \quad (10)$$

Relation (10) enables us to determine a minimum rotary speed at which the contact between the sensor spindle tip and the object will be lost, depending on the object diameter and the nature of changes of the measured profile and measuring pressure  $P_k = F_{sp}$ .

### Testing the model

Based on relation (10), an analysis was made to find out how the measured object diameter and parameters describing the measured roundness profile and acceleration,  $a_{o1}$ , affect the value of minimum rotary speed at which the spindle tip-object contact will be lost. Calculations were made for regular roundness profiles described by relation (8), for  $n = (2 \div 45)$ , which are the describing the harmonic profiles of the shape and  $n = (60 \div 480)$  describing the profiles of waviness. The analysis involved an object with a diameter  $D_o = (0 \div 300)$  mm and measuring pressure  $P_k = (0 \div 0.96)$  N (measuring tip mass,  $m = 4$  g). The test results are shown in charts quantitatively and qualitatively illustrating the dependence between the factors under consideration. Example charts are given in Figure 2 (a–h).

The calculations have shown that a change in diameter,  $D_o$ , does not significantly affect the minimum value of rotary speed at which the measuring tip may lose contact with the measured profile. The influence of this parameter is visible only for small diameters  $D_o = (0 \div 4)$  mm. The decisive impact comes from the shape of the measured profile described by parameters  $C_n$ ,  $n$ , and measuring pressure  $P_k$ .

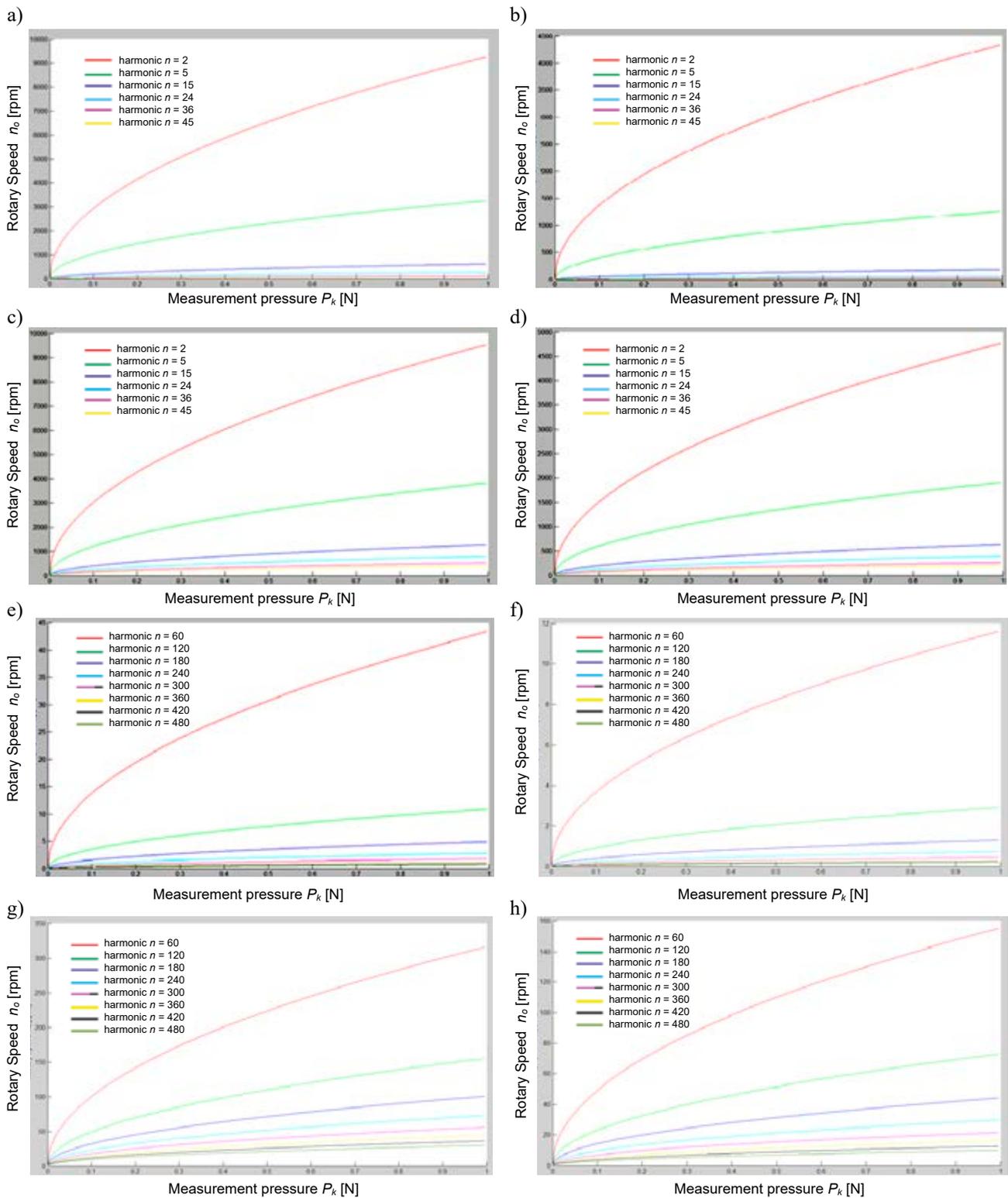
The mean measuring pressure of inductive sensors presently used for measurements of shape deviations and profiles is  $P_{ki} = 0.63$  N, which corresponds to an acceleration  $a_{o1i} = 157.5$  m/s<sup>2</sup>. The rotary speed range applied in measurements of roundness profiles

should not exceed a few revolutions per minute (especially for large parameters  $C_n$  and  $n$  characterizing their profile). Inductive sensors with a movable spindle have a measuring pressure not higher than 0.5 N. In such a case, the probability of contact loss is high. The probability substantially rises for small diameters of the measured object, as illustrated in Figure 2e and f.

However, if we assume that assessment will comprise shape deviations described by harmonics in the range  $n = (2 \div 45)$  and that the recommended rotary speeds should not exceed 6 rev/min in this case, then maintaining an average measuring pressure  $P_k = (0.5 \div 0.65)$  N will have solid grounds to expect that the measuring tip will not lose contact with the measured profile surface.

Such a conclusion is based on the results of the model simulations of force distribution at the point of contact between the spindle tip and the measured object. The simulations were executed using the Working Model 2005 program for modelling the actual roundness profile of the measured object. The profile was obtained by measuring the roundness profiles of main journals of a crankshaft whose extreme journals were set in V-blocks. Recorded analogue signals were discretized into digital signals, allowing the data to be presented mathematically and graphically as charts in either polar or Cartesian coordinate systems. The graphical representation of the profile provided a basis for analysis of the forces acting at the contact of the measuring tip and the measured profile using the aforementioned Working Model 2005 program. In simulation tests, the varied parameters included the profile's rotary speed, measuring tip pressure and parameters of the measuring system. For the examined profile described by  $n = 50$  harmonics and a roundness deviation of 48.9  $\mu\text{m}$ , with the actual proportions of the measuring system parameters maintained, the standard measuring tip pressure, and the rotary speed ranging from 0 to 8 rev/min, the forces at the measuring tip-measured profile contact point were found not to compensate each other to zero.

The analytical-simulation tests were followed by experiments. These included measurements of shaft external surface profiles with repeated irregularities. A specimen specifically prepared for this purpose was a shaft section with a 300 mm diameter whose external central part had a series of grooves made at regular intervals and with blunt edges. There were 360 grooves of 150  $\mu\text{m}$  deep symmetrically distributed on the shaft circumference. Cross-sections were measured on the shaft while its external



**Figure 2.** Graphic interpretation of relation  $n_o = f(P_k)$  for: a)  $C_n = 50 \mu\text{m}$ ,  $R_o = 2 \text{ mm}$ ,  $n = \langle 2 \div 45 \rangle$ , b)  $C_n = 200 \mu\text{m}$ ,  $R_o = 2 \text{ mm}$ ,  $n = \langle 2 \div 45 \rangle$ , c)  $C_n = 50 \mu\text{m}$ ,  $R_o = 150 \text{ mm}$ ,  $n = \langle 2 \div 45 \rangle$ , d)  $C_n = 200 \mu\text{m}$ ,  $R_o = 150 \text{ mm}$ ,  $n = \langle 2 \div 45 \rangle$ , e)  $C_n = 50 \mu\text{m}$ ,  $R_o = 2 \text{ mm}$ ,  $n = \langle 60 \div 480 \rangle$ , f)  $C_n = 200 \mu\text{m}$ ,  $R_o = 2 \text{ mm}$ ,  $n = \langle 60 \div 480 \rangle$ , g)  $C_n = 50 \mu\text{m}$ ,  $R_o = 150 \text{ mm}$ ,  $n = \langle 60 \div 480 \rangle$ , h)  $C_n = 200 \mu\text{m}$ ,  $R_o = 150 \text{ mm}$ ,  $n = \langle 60 \div 480 \rangle$

end surfaces were mounted in center points. The measurements were made by changing the rotary speed of the shaft and the pressure of the measuring tip. During these measurements, the shaft speed

was varied smoothly, while the measuring spindle axis ran horizontally. In this way, the weight of the spindle did not affect changes of the measuring tip pressure.

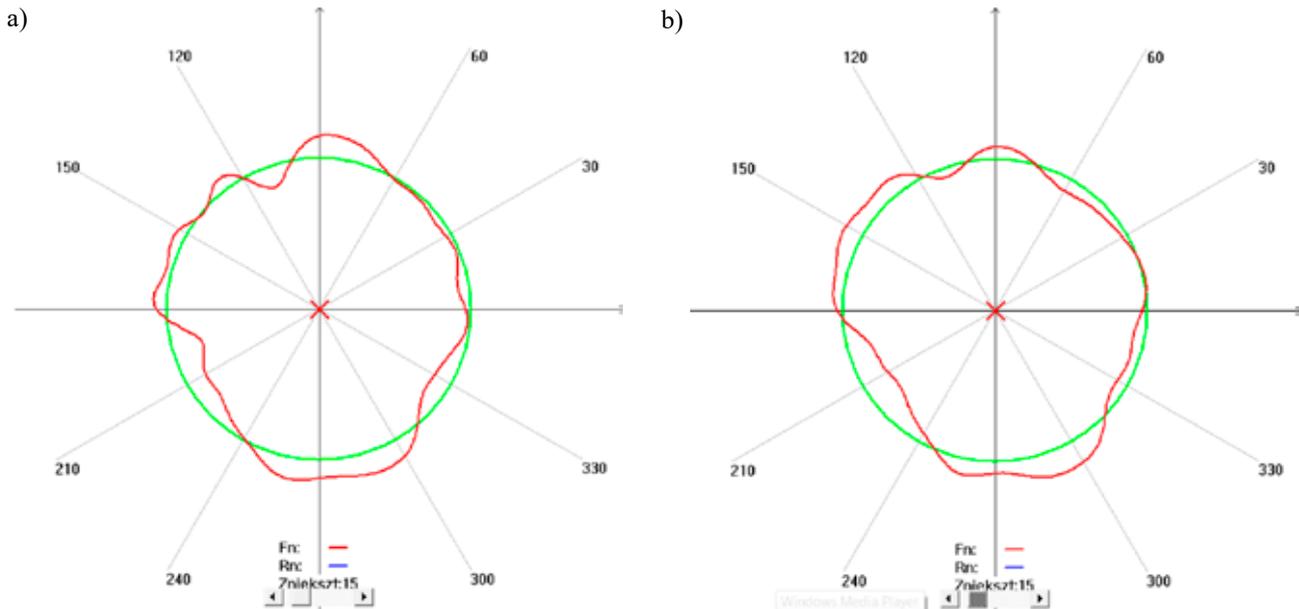


Figure 3. a) Measured roundness profiles for measuring pressure 0.5 N and rotary speed 6 rev/min, and b) measuring pressure 0.5 N and rotary speed 12 rev/min

The assessment was made by comparing specimen profile measurements obtained for various rotary speeds. The results confirmed previous observations that there is no risk of losing contact between the measuring tip and the measured profile at low rotary speeds. However, an increase in the rotary speed at a minimum tip pressure leads to a gradual rise in the amplitudes of spindle displacements, yielding essential changes in the shape of the measured profile. A similar conclusion can be drawn from results of measurements of actually irregular roundness profiles encountered in practice. In this case the measured item was a set of marine engine crankshaft journals. Example measurement results of roundness profiles of journal No. 3 obtained for varied rotary speeds and constant measuring tip pressure 0.5 N are shown in Figure 3a and b. The measured profile was characterized by significant irregularities, and as a result, at  $n_o = 12$  rev/min essential changes were observed in the shape of the profile and there was a consequent increase in the roundness of deviation.

## Conclusions

The presented test results indicate that in measurements of cylindrical surface roundness profiles and deviations the rotary speed of the measured object should not exceed 10 rev/min. This conclusion is based on theoretical considerations, simulation tests and experiments. The results of these tests have also shown that an increase in rotary speed

directly affects the shape of the obtained profile and the determined value of roundness deviation. Whether contact will be lost between the measuring tip and the object depends largely on the actual shape and character of irregularities of the measured profile. Therefore, the rotary speed should be adjusted to the roundness profile being measured, and such adjustment may be based on analytical relations between the object rotary speed and parameters describing the shape of the measured profile.

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