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Optimization approach in multi-stop routing of small islands

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Abstract

The routing problem of small island ports is, in many cases, firmly dependent on country topology, e.g., how to connect islands with a main (home) port, where the order of stops can be different, especially if there are not enough passengers or cargo waiting to be transported to or from every port. Thus, we need a capable optimization tool with which we can adapt each route for an appropriate time schedule; for example, some routes in one cycle can touch each island (forwards or backwards) but some routes can be incomplete, to touch only a few of them. The carrier has to find space for price-cutting (lower prices per journey – more passengers on board), to be more attractive in free-market competition. In such route optimization, we have to interconnect minimal transport cost with maximal revenue (money from tickets), which could be a very demanding task (a non-linear objective cost function). Instead of a non-linear polynomial optimization, which can be very complicated and time-consuming, the network optimization methodology could be efficiently applied. The main goal is to find more efficient routes, to decrease expenses and to increase revenue at the same time (dual mini/max problem).

Introduction

Connecting small islands by ship (ferries), or even by plane, is a familiar problem, whether we refer to daily schedules of trips or multiple trips by one ship per day, whether by a small boat or a large liner. This scientific research does not consider connection of bigger islands with huge amounts of traffic; such islands are connected with the home port directly and, for such routes, there is no space for innovation and route optimization. At the moment, however, in many EU countries (e.g., Republic of Croatia), there is only one carrier that connects small islands in one shipping route, but soon this may change and competing carriers may appear.

European Union regulations allow the state to attract foreign carriers to compete in the market (division of concessions). When this happens, the domestic carrier needs to be concerned about its profitability and how to be more competitive in the market. Usually, shipping routes that connect small island ports tend to be unprofitable, because only a small number of people live and work on those islands. Some of the parameters that can determine the profitability of the route are revenue from travel tickets, travel costs (fuel consumption), and the cost of loading/unloading of cargo, passengers or vehicles, as well as entering a port, which can be a significant cost. In a situation where not all would-be passengers are able to embark on a ship in a certain port, then there will be space for another carrier. Mutual cooperation between carriers can be developed and may even include the coordination of routes, in order to satisfy demand more efficiently. There is always a possibility of mutual alignment of routes, to avoid competition of carriers around each passenger, vehicle, etc.

Passenger or cargo consignments have defined starting and ending points of shipping and their inter-relation is obvious (as long as they are on the same ship). Quantities of different consignments of cargo with a specific starting or destination port are closely related to each other because the total capacity of the ship is limited. Normally, we have one, two or more consignments on board. Taking into consideration traffic demands for loading/unloading of each consignment (including passengers or vehicles), and for each port where they are waiting to be transported, we have to search for an optimal transportation plan that will minimize transportation costs, costs of loading/unloading and waiting of the ship in ports, in order to have a profitable route for the carrier. A tool which can devise such plans can significantly facilitate the comparison of possible routes for an individual ship, or for different types of ship with different capacities, if a choice is available.

The problem of optimal transportation for multiple loading ports and unloading ports with $m = 1,2,\ldots,M,$ and multiple consignments of cargo $i = 1,2,\ldots,N$ is very difficult. However, it can be solved with different techniques (Yan et al., 2007; Xiea & Jiab, 2012). Of course, we are trying to find an acceptable heuristic algorithm that gives a satisfactory solution with a significant reduction in the complexity of the calculation.

This optimal transportation problem can be seen as a special case of the problem of multi-commodity flows through a network – Minimum Cost Multi-Commodity Flow Problem (MCMCF); see the theoretical background in Castro and Nabona (Castro & Nabona, 1996) and Zangwill (Zangwill, 1968). Some applications of similar approaches can be seen in Krile (Krile, 2013a) and Krile et al. (Krile, Krile & Prusa, 2015).

In this article, we apply such an approach to network optimization, with certain modifications; for example, in determination of possible capacitive states with the help of a combinatory formula. In this paper, for test-examples, we use a minimal capacity increment of 10%, so the problem of complexity is within acceptable limits for average computing power. Such an approach is also capable of solving non-linear transportation problems.

In the first section, we explain the transportation problem solved by a network optimization approach. The mathematical model for the routing problem is explained and a heuristic algorithm development is shown. The next section describes different configurations of islands and types of routes. After this, we describe the optimization tool for finding an adaptive route. In the last section, we discuss the results of the proposed optimization tool, on the basis of simple test-examples.

Optimization approach

Different cargo consignments (of passengers or vehicles) are differentiated with i for $i = 1,2,\ldots,N$. A ship with a defined capacity ships from the first to the last port marked $M+1$, with a possible set of intermediate ports marked K. The amounts of different consignments are in fixed correlation because the total capacity of the ship is limited. The objective is to find a loading/unloading and transportation strategy that minimizes the total cost incurred over the whole route, consisting of M ports on the path ($M \leq K$). The starting port on the route can be only for loading and the last port on the route can be only for unloading; other ports on the route may be for both.

The transportation technique can be seen as a capacity expansion problem (CEP). Transportation segments (ship space) are capable of serving N different cargo consignments, e.g., passenger or cargo loads (multi-commodity) for $i = 1,2,\ldots,N$; see elaboration in Fleisher (Fleisher, 2000). A new capacity segment on board the ship can be assigned to the appropriate load, up to the given limit (maximal capacity). The used capacity can be dimensioned in two forms: by expansion ($x_{i,m}$) or by reduction ($r_{i,m}$).

Expansions and reductions can be done separately for each consignment loaded or unloaded. Figure 1 gives an example of the network flow representation for multiple consignments (N) and M ports on the route. The port marked with $M+1$ represents the home port. Thus, the transportation problem can be represented by a flow diagram of an oriented acyclic network.

Let G (V, E) denote a network topology, where V is the set of vertices/nodes, representing capacity

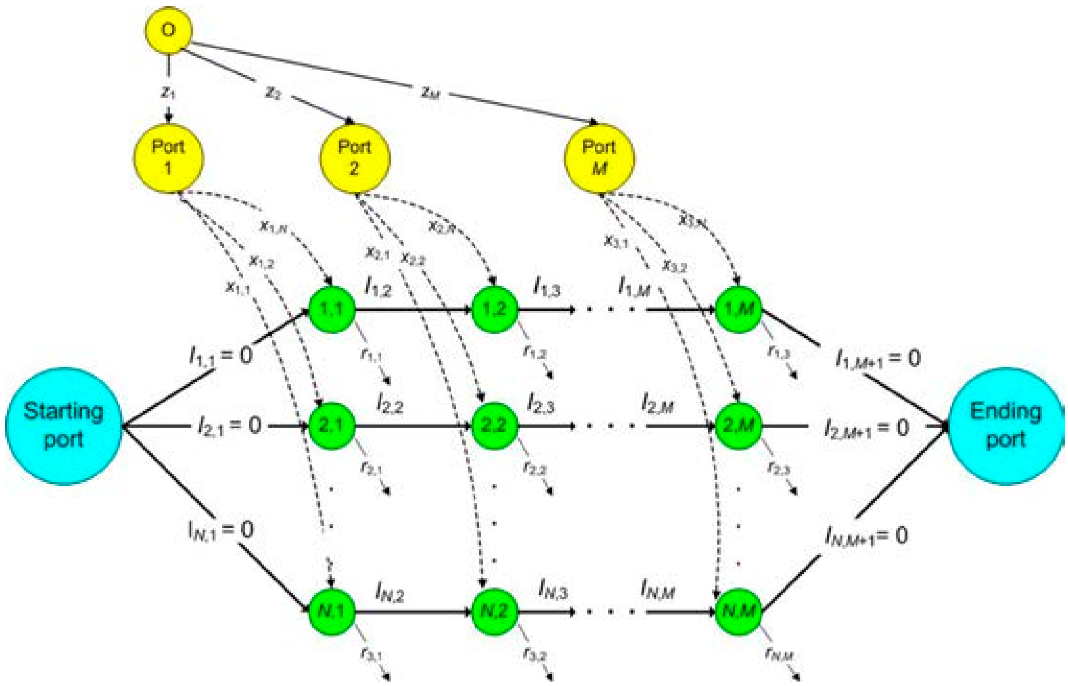

Figure 1. The transportation problem can be represented as a CEP problem and can be solved as the shortest path problem for an acyclic network in which the nodes represent all possible values of capacity points. The links connecting neighbor ports on the route represent cost values

states on the board and A, the set of arcs (links) representing traffic changes (loading/unloading, transportation, port services etc.) between ports. Each link on the route is characterized by a z-dimensional link weight vector, consisting of z-non-negative weights. In general, we have a multi-constrained problem (MCP) with multi-dimensional link weight vectors for M+1 links on the path $\{w_{i,m}, m \in A, i = 1,\ldots,N\}$. The constraints for capacity bounds are denoted by $L_{i,m}$ ($L_{1,m}$ $L_{2,m}$, …, $L_{N,m}$). The total capacity of the ship is the same at any point of the route.

For an additive measure (load of passengers), the definition of the constrained problem is to find a path from the starting to the end port with minimal weight to satisfy maximal traffic load. This is equivalent to minimal cost; that is, the function of all expenses. A shorter distance gives lower weight. Also, the weight of each link corresponds to the amount of capacity used. Also, more load on board causes lower transportation cost per load unit (e.g., passenger or vehicle).

The objective is to find the optimal routing/loading sequence that minimizes the total cost with maximal cargo on board. In the context of MCP, we can easily introduce the adding constraints e.g., maximal length of the route.

The main reason for our approach is the possibility of discrete capacity values for a limited number of consignment loads, so the optimization process can be significantly improved.

The definition of the single-constrained problem is to find a path P from starting port to end port, such that:

$$w(P) = \min \sum_{m=1}^{M+1} \sum_{i=1}^{N} w_{i,m}(I_{i,m}, x_{i,m}, r_{i,m}) \qquad (1)$$

where: $$I_{i,m} \le L_{i,m} \qquad (2)$$

for $i = 1,\ldots,N$; $m = 1,\ldots,M$.

A path obeying the above conditions is said to be feasible. Note that there may be multiple feasible paths between starting and ending port.

Generalizing the concept of the capacity states after loading/unloading each consignment m between ports on the route, we define as *a capacity point* – α_m.

$$\alpha_m = (I_{1,m}, I_{2,m}, \ldots, I_{N,m}) \qquad (3)$$

$$\alpha_1 = \alpha_{M+1} = (0, 0, \ldots, 0) \qquad (4)$$

In formulation (3) α_m denotes the vector of capacities $I_{i,m}$ for each load i and for each port m, and we call it a capacity point. With z_m we denote the total loading amount in the port m. On the flow diagram, Figure 1, each column represents a capacity point, consisting of N capacity state values (for i-th cargo load). Formulation (4) implies that zero values are the same before loading at the starting point as after unloading at the ending point. Let C_m be the number of possible capacity point values at port m. Only one capacity point is for starting and only one for the end port on the route: $C_1 = C_{M+1} = 1$. The total number of capacity points is:

$$C_p = \sum_{m=1}^{M+1} C_m \tag{5}$$

On the diagram, horizontal links (branches) represent the capacity flow $I_{i,m}$ between two neighboring ports on the route.

The network optimization can be divided into two steps. In the first step, the minimal transportation weights $d_{u,v}$ between all pairs of capacity points (neighboring ports on the route) are calculated. It is obvious that, in CEP, we have to find many cost values $d_{u,v}(\alpha_u, \alpha_{v+1})$ that emanate from the two capacity points of neighboring ports.

Most of the computational effort is spent on computing these values, depending on the total number of capacity points, see (5). The total number of all possible $d_{u,v}$ (α_u, α_{v+1}) values representing the CES (Capacity Expansion Sub-problem) between two capacity points is:

$$N_d = \sum_{m=1}^{M} C_m \cdot C_{m+1} \tag{6}$$

In the second step, we are looking for the shortest path in the network with the previously calculated weights; that is, the transportation cost $d_{u,v}$ (α_u, α_{v+1}). Then Dijkstra's or Floyd's algorithm, or any similar algorithm for the shortest path calculation, can be applied; see Zangwill (Zangwill, 1968) or Foster (Foster, 1995). An overview of techniques for convex optimization can be seen in the paper by Ouorou et al. (Ouorou, Mahey & Vial, 2000).

As the number of all possible $d_{u,v}$ (α_u, α_{v+1}) values depends on the total number of capacity points, it is very important to limit that number (C_p), and this can be done through imposing appropriate capacity bounds or by introducing adding constraints (e.g., minimal capacity increment, e.g., 10% in our test-example). Through numerical test-examples, we will see that many loading/unloading solutions cannot be a part of the optimal expansion sequence. Thus, we can obtain the near-optimal result with significant computational savings. Similar problems are solved in papers by Krile (Krile, 2011; 2013b).

Possible configuration of islands and different types of ship routes

First of all, we need to distinguish two types of routes: routes that are naturally defined and those which can be constructed by optimization of parameters, such as the distance between islands and home port, with the aim of finding the shortest route possible. The first type of route is used when we want to connect the main port (H – home port) with islands positioned in series, see Figure 2b. For example, such a situation applies in connecting the city of Dubrovnik with the Elaphite Islands (Koločep, Lopud, Šipan). Today, those lines are implemented in such a way that, after a certain waiting time in the final port (half cycle), return from the last to the first port is performed in the reverse order (similar path – the shortest route).

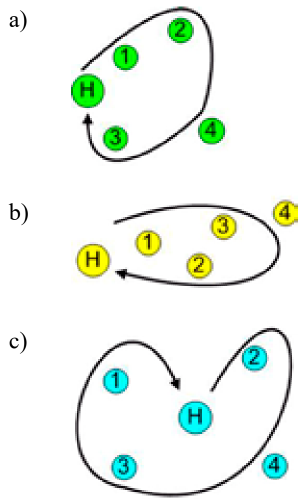

Figure 2. Different port configurations in relation to starting port (home port)

But in many other configuration schemes, a full-cycle route is not operable because of the small number of passengers. Sometimes, it is not necessary to stop in every port, which depends on the number of passengers, but this has to be announced in advance in the timetable. Today, it is not possible to skip some ports on a route because of the lack of information about the exact number of passengers and cargo consignments waiting to be transported, till you stop in the port.

One-way routes or half cycles (with long waiting time for return) are possible in a situation when travelers go from the home port (H) to islands or vice versa. For example, they may start early in the morning and come back from the island in the afternoon or evening. It could be extended to a few times a day maximum. This means that passengers do not travel so often and they are satisfied with one-way lines. One-way routes (one direction) are especially common in winter time. It is clear that return to the starting port immediately (closing the circle), significantly increases the transport cost.

A unidirectional route is a big restriction and the norm should be bi-directional routes, which means that every island should be reachable from any other destination, with equal communication possibilities. It will lead to the appearance of new contingents of passengers who want to reach a certain island at any moment.

In further research, we use a simple example to provide an overview of problems on the routes with a higher number of ports and a higher number of consignments of passengers or cargo. For our test-example, we have only 4 (four) ports, but the number of possible consignments grows to 12: 1-2, 1-3, 1-4, 2-3, 2-4, 3-4, 4-3, 4-2, 4-1, 3-2, 3-1, 2-1. If we have vehicles on board it could be doubled. In our example, from Figures 3 and 4 (with only 4 ports) we have 7 consignments in total, which is a relatively small problem.

Nowadays, a very important type of passenger is a tourist, moving in all possible directions. Currently, tourists are limited in their traffic demands. In the sense of multi-stop routes, there will be a need for a shipping route to become bi-directional, where islands can intercommunicate more often and fully.

In the other two route cases, see Figure 2a or 2c, it is necessary to find an optimal one-way route. But in these types of routes, each port is touched only once. The current mode of transportation is a one-way route, which is very good for fuel savings, but

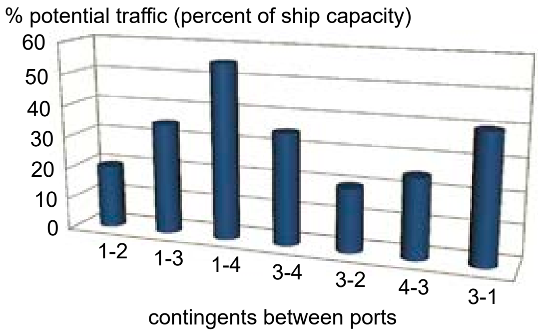

Figure 3. Test-example: Potential traffic per contingent (between certain ports)

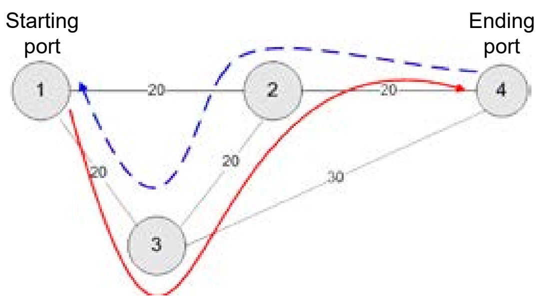

Figure 4. Test-example: Position of ports and their distances. The route does not have to be equal in both directions. Port 2 can be skipped if traffic to/from port 2 is small

there are communication limitations. Figure 2b is a special case, where the cycle finishes with returning to the home port (the longest route), but all ports can be touched twice. In previously mentioned cases (type 2a and 2c), the objectives can be achieved only in two subsequent cycles, with a possible short waiting time in home port.

Possibility of adaptive routes

Potential cargo information is mainly obtained from statistical data, which is sometimes insufficient. Nevertheless, today's ICT technology is able to provide information about current demand for transportation from island to the island, which means that we can know exact information about potential cargo that is waiting in some port to be transported to another port. With this in mind, this paper considers some new possibilities, primarily through dynamic routing, which should follow requests for transport, and not just use existing lines on the schedule. Mainly, this is possible due to an increase in the frequency of visiting ports, because visits can be maximally doubled (backward and forward).

Of course, the extreme situation is when all ports have to be visited constantly, which is feasible only if the cargo (passengers or vehicles) is sufficient to make profit in every port, and with high route frequency. In practice, the cost functions are non-linear, continuous or non-continuous, which shows the effect of economy of scale. They consist of fixed and variable costs, depending on the quantities (distance, number of passengers, cargo, time, various fees etc.). In our test-example, we use an exponential function shape with a_{im} – coefficient of concavity for each type of cargo and for certain transportation conditions in each port on a shipping route.

$$c = A_{i,m} + B_{i,m} \cdot I_{i,m}{}^{a_{i,m}} \tag{7}$$

Revenue (income) is expressed with c_{freight} for a single consignment. $I_{i.m}$; c_m is the cost of cargo transportation caused by distance; h_m cost of loading ($x_{i,m}$)/unloading ($r_{i,m}$) + staying in the port m. Target (objective) function can be formulated as a profit maximization, but also as a transport cost minimization, which is then revenue added with negative polarity, so the function would look like:

$$\min \sum_{m=1}^{M-1} \left\{ c_m(d_m) + h_m(x_{i.m} + r_{i,m}) - c_{\text{freight}}(I_{i,m}) \right\} \tag{8}$$

where: $$I_{i,m+1} = I_{i,m} + x_{i,m} - r_{i,m} \tag{9}$$

We need to emphasize that cyclic routes in nature are unidirectional, if every port is visited only once, but in the next cycle this will be corrected, and all transportation needs can be satisfied (all possible directions of travel). It means that we need two half-cycles. For example, from Figure 4, if we want to travel from port 3 to port 2, we cannot do it in return, and need to wait in the home port for the next cycle.

As we said before, for a large amount of traffic, it can be solved with two ships, operating in both directions simultaneously. Quantities of potential passengers and vehicles waiting in the port can easily be expressed as a percentage of total ship capacity.

For small amounts of traffic, some ports can be skipped. Complete routes with double cycles are the best solution, but it could be the most expensive solution. A passenger often has to pay twice, in each direction separately, especially if the other direction (on return) is not immediately on schedule. If we are introducing the complete route (both directions), such an approach should enable the same price for any combination of starting/ending ports, no matter in which direction you travel/cycle (the first or the second).

Results of optimization

In Figure 3, we can see input traffic demands among 4 ports for the port topology shown in Figure 4. We can see passenger (or cargo) consignments for a certain direction, expressed in a percentage of the total capacity of the ship. Figure 4 shows distances between two ports in miles. It is obvious that port 2 is outside the main direction of shipping toward the last port, port 4. Therefore, visiting that port can cause significant additional cost. When there is insufficient cargo waiting to be transported to/from port 2, the optimal route should be: 1-3-4-3-1; see Figure 5. This diagram shows the optimal route solution with reduced transport cost because of the smaller number of visited ports (skipping port 2). The tendency is that the carrier stops in only those ports that make profit, and especially avoids those ports where there is not sufficient incoming or outgoing traffic. This means that market rules sometimes justify the trend: "for less work – much more profit". But it means that some passengers are still waiting to be transported, possibly with another carrier. Of course, in such a situation, other carriers could appear, offering the same or better service for a lower price. In this test-example, we keep all price elements equal, so we concentrate on occupancy of the ship as the measure of route efficiency.

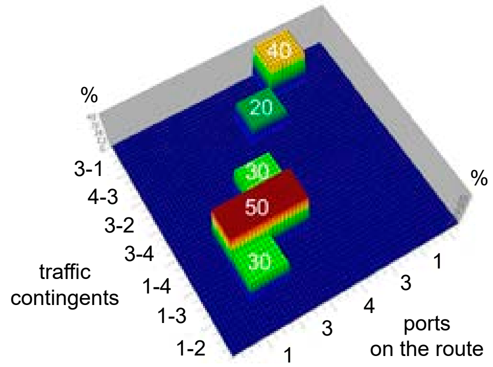

Figure 5. Amount of cargo load per consignment between certain ports, as a result of optimal route. Port 2 is skipped because of insufficient traffic

Sometimes, the service quality (higher speed) or transport comfort (nice interior) can also be an important element. The reduction of ticket prices could play the key role in free-market competition. Some travelers use old and slow ships but oversized in capacity so that they always accept all passengers and cargo on board. Today's ships are smaller and much faster (catamarans, etc.) but they have limited capacity, which should be taken into account. It is possible to express these values as the number of passengers or vehicles during the voyage but expression as a percentage of total occupancy of the ship could be more efficient; see Figures 5–8.

Now we want to show what will happen if we increase the amount of traffic to/from ports we skipped in the previous example. If significant traffic exists from/to port 2, the optimal route should be: 1-2-3-4-3-1 or 1-3-4-3-2-1. In our test-example, we increased the amount of traffic to port 2, e.g., contingent 1-2 (for more than 20%) and/or contingent 3-2 (for more than 20%). In that case, the optimal route is complete: 1-2-3-4-3-2-1, which means that the route is profitable in every segment; see Figure 7. We can nicely track the amount of cargo load on board between each port and for each contingent. Figure 8 shows total occupancy (load) of the ship. If we compare what happened, we can see the increase of ship occupancy in the first half of the cycle (near 100%), but on return it is slightly more than 40% on average. However, it is also much higher than before (see Figure 6).

Of course, the transportation price (travel ticket) could significantly affect these results, as could other cost elements (e.g., port taxes). As we said before; in this test-example, we keep all price/cost elements equal. Normally, the cost elements, including ticket price, influence optimization results very strongly.

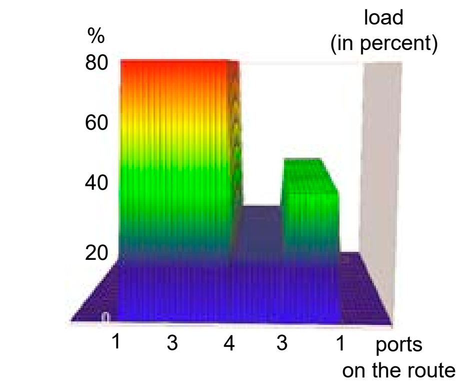

Figure 6. Small occupancy of the ship because of skipping the port 2

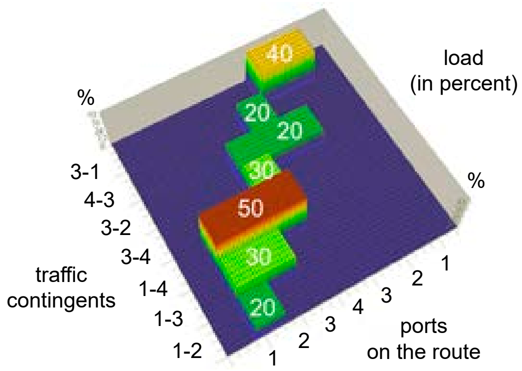

Figure 7. New route that includes port 2 because of increased amount of cargo, both in departure (contingent 1-2) and in return (contingent 3-2)

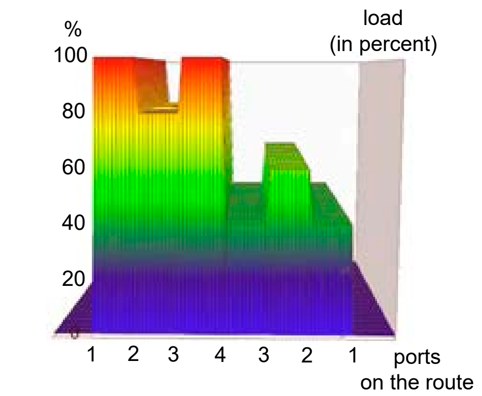

Figure 8. Occupancy of the ship on the route is much better than before

Conclusions

Route optimization should reduce the cost of transportation significantly, especially with the possibility to skip ports with small traffic. Introduction of complete, double-cycle routes will increase the possibility of transportation of all passenger/cargo consignments, because all possible requests could be satisfied. In that way, the traffic will rise and profit should increase. In high traffic demands, it would be the best approach to introduce complete routes. With increased traffic, e.g., in the tourist season, more carriers could operate in parallel, because the size of the ship, or introduction of a new ship, is the last thing that could be changed. With higher traffic, the occupancy of ships will increase, as the measure of transport efficiency.

This research is assumes the presence of several carriers transporting simultaneously. Carriers could divide routes during the day or even by days in a week; for example, alternate days. Also, they can create adaptive routes within one day. We must not exclude the need for increased numbers of ships or bigger ships. With the presented optimization tool, the carrier can easily estimate what kind of ship is most suitable for a particular traffic amount on a route with many small ports (e.g., small islands). Such a routing approach could increase the frequency of passenger travelling, and shorten the waiting time to return to the home port, which should certainly increase passenger satisfaction.

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