

Modeling vessel passage speed by using passage-time in restricted areas

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Abstract

Vessel passage speed is one of the parameters describing the vessel traffic stream on a selected waterway. Knowing the probability distribution of vessel passage speeds is essential for modeling vessel traffic streams on a waterway. This article undertakes probabilistic modeling for vessel speeds in restricted areas, where the distribution of the vessel passage time of the waterway section is known. The probabilistic procedure of the inverse random variable is used. Four different cases are considered. First, the probabilistic distribution of the vessel passage speed is given, where the vessel passage time is described by the normal distribution in certain restricted areas. The next three cases present the probabilistic distribution of vessel passage speeds on the Szczecin–Świnoujście fairway, where the vessel passage time is described by the extreme value distribution, the Fréchet distribution and the Weibull distribution.

Introduction

Over the years, the vessel traffic density in water areas has increased significantly, as economic considerations led to increased interest in maritime transport. It is, therefore, reasonable to examine vessel traffic processes and these processes are widely studied. The analysis of vessel traffic flows is one stage of research that has been conducted (e.g., Xu et al., 2013; Majzner, 2015; Wen et al., 2015; Weng & Xue, 2015; Yoo, Jeong & Yim, 2015; Liu et al., 2017). In turn, this research can be used in simulated transport system studies, in analyzing the relationships between vessel streams, in analyzing vessel passage over underwater objects, e.g., pipelines, or modeling a vessel collision with a platform or another vessel in an oil field. These processes are analyzed on different waterways and studied in order to model them with high accuracy.

Many solutions to problems of marine traffic engineering require knowledge of probability distributions of different random variables; among others, the vessel passage time of a waterway section and the vessel passage speed of this section. This is a very important issue, especially in order to perform computer simulations using the simulation models, which have been developed (Gucma, Gućma & Zalewski, 2008; Liu, Fu & Cong, 2013; Hou et al., 2014). Both the vessel passage time and the vessel passage speed, calculated from fitted distributions, can be applied as input data in such traffic simulation models. These simulations allow researchers to carry out basic studies, but also to evaluate real vessel traffic processes. It is worth noting that such simulated vessel traffic stream models are used in order to evaluate shipping safety and improve navigational traffic on the selected waterway sections.

Normal distribution is the most frequently used probabilistic model for vessel speeds on

a fairway section where there are no disruptions (Gucma & Schefs, 2007). However, certain regulations are applied, or disruptions might occur on different fairways. For example, there are special requirements of vessel traffic on the Szczecin–Świnoujście fairway in Poland: vessel passage or overtaking is limited; vessel speeds are restricted on selected fairway sections; a minimum distance must be maintained between successive vessels, etc. Therefore, on such fairways the vessel speed will not be normally distributed (Kasyk, 2012; Kasyk & Kijewska, 2013).

In addition, some marine traffic monitoring systems or vessel registration systems, e.g., VTS, only register the vessel report at a certain point. Therefore, only a data set describing the vessel's entry time is obtained. However, this allows the vessel's passage time on a selected waterway to be calculated and, on the basis of the vessel passage time, one can calculate the vessel's passage speed.

In this article, the authors propose to build a probabilistic model for vessel speeds on a certain waterway section by taking this random variable as a function of another random variable – the vessel passage time on the selected waterway section. Thus, the authors assume that the probabilistic model for vessel passage time on the selected waterway section is known and the functional relation between vessel passage speed and passage time on this waterway section is also established. This article presents a probabilistic model for vessel passage speeds on selected waterway sections as a result of the approach proposed by the authors.

The remainder of this article is organized as follows. In the second section, an approach to modeling the vessel passage speed in certain restricted areas is presented. This approach is based on the inverse random variable procedure. In the third section, this approach for modeling vessel passage speed is applied, but vessel passage time on a selected waterway section is described probabilistically as a random variable with normal distribution, determined in the standard way. In the fourth section, the authors carry out a case study for the probabilistic modeling of passage speeds for different vessel types on the section *KARSIBOR – DOK5* of the Szczecin–Świnoujście fairway. The final section expresses conclusions.

Approach for modeling the vessel passage speed

The function of one random variable is the basis for the presented approach (Kasyk, 2012). If X is

a given random variable with the probability density function $f(x)$ and Y is a function of the random variable X , denoted by $\varphi(X)$, then the probability density function of the random variable Y is as follows (Kasyk, 2012):

$$g(y) = f(\varphi^{-1}(y)) \left| \frac{d\varphi^{-1}}{dy} \right| \quad (1)$$

In addition, one should assume that the function φ is injection.

One of the ways to determine vessel speed is the measurement of the passage time on a selected waterway section. One of the possible functional relations between the vessel passage speed and its passage time on that section is as follows:

$$v = \frac{d}{t} \quad (2)$$

where:

d – waterway section length;

t – vessel passage time on the waterway section.

In the random variable language, formula (2) is given as follows:

$$V = \frac{d}{T} \quad (3)$$

where:

V – random variable denoting the vessel passage speed on the selected waterway section;

T – random variable denoting the vessel passage time on this waterway section.

Knowing the probability distribution for the vessel passage time T on the selected waterway section and using formulas (1) and (3), one can model the inverse random variable, that is, the vessel passage speed V . In consequence, the probability density function g of the vessel passage speed V on this waterway section is as follows:

$$g(v) = \frac{d}{v^2} \cdot f\left(\frac{d}{v}\right) \quad (4)$$

where:

d – waterway section length;

f – probability density function of T .

Formula (4) describes the relation between the probability density function g of vessel passage speed V and the probability density function f of vessel passage time T on the selected waterway section.

Modeling the vessel passage speed by using the standard probabilistic models for its passage time

The vessel passage time as a normally distributed random variable

One of the most frequently used probabilistic models for describing the vessel passage time is the normal distribution. This model is applied, e.g., in selected restricted areas, when there is no disruption or the vessel traffic flow is not too much disturbed.

Suppose, then, that the vessel passage time T has a normal distribution with rates μ and σ . Since T cannot be 0, then the formal problem is introduced. Therefore, a truncated distribution can be applied. The probability density for the random normal variable after restricting its range to be greater than 0, is as follows (Kasyk, 2012):

$$f(x) = \frac{2}{\left(1 + \operatorname{erf}\left(\frac{\mu}{\sigma\sqrt{2}}\right)\right) \sigma\sqrt{2\pi}} \cdot \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right) \quad (5)$$

for $x > 0$. For $x \leq 0$, $f(x) = 0$. The function $\operatorname{erf}(x)$ denotes the error function, which is defined as follows:

$$\operatorname{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x \exp(-t^2) dt \quad (6)$$

By (4) and (5), one can obtain the probability density function g for vessel passage speed V on the selected waterway section at the length d , only for positive speeds v :

$$g(v) = \frac{2d}{\left(1 + \operatorname{erf}\left(\frac{\mu}{\sigma\sqrt{2}}\right)\right) v^2 \sigma\sqrt{2\pi}} \cdot \exp\left(-\frac{\left(\frac{d}{v} - \mu\right)^2}{2\sigma^2}\right) \quad (7)$$

where vessel passage time T has a normal distribution with rates μ and σ . For $x \leq 0$, the value of this function g and the below probability density functions of the vessel passage speed equal 0.

The graph of the probability density function $g(v)$ for vessel passage speed V is depicted in Figure 1. To illustrate the shape of the probability density function described by formula (7), suppose that vessels pass a 5 Nm-long waterway section in an average time of 0.35 hour, with the standard deviation 0.05 hour ($d = 5$ Nm, $\mu = 0.35$ h, $\sigma = 0.05$ h). One can

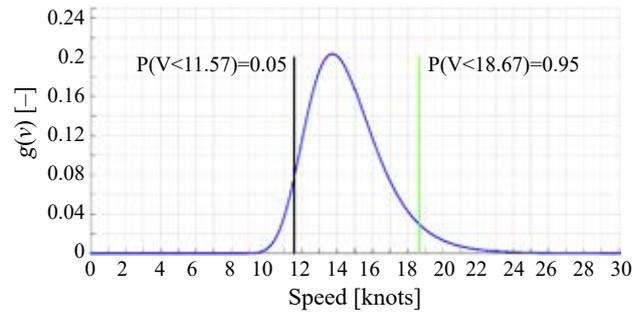


Figure 1. The probability density function $g(v)$ for the vessel passage speed, where vessel passage time is normally distributed. Case study: modeling vessel passage speed on the selected Szczecin–Świnoujście fairway section

observe that the obtained random distribution of the vessel passage speed V has a positive coefficient of skewness (long right tail). One could prove that, for any positive coefficients d , μ , σ , the graph has the same property. Moreover, one can observe that the probability that the vessel passage speed is less than 11.57 knots equals 0.05 and the probability that the vessel passage speed is less than 18.67 knots equals 0.95. On such a waterway, the average value of vessel passage speed V equals 14.59 knots. Moreover, its standard deviation equals 2.24 knots.

The Szczecin–Świnoujście fairway is a sea waterway running from Pomeranian Bay through Świna, Mieliński Channel, Szczecin Lagoon, Odra River, and Przekop Mieleński to Szczecin Port. This fairway is 68 km long. The Szczecin–Świnoujście fairway is covered by many regulations; among others, vessel passage, overtaking and vessel speeds are limited; a minimum distance must be maintained between successive vessels; there are passage bans, etc. Consequently, the vessel traffic stream on this fairway is disturbed.

In this part of the research, the Szczecin–Świnoujście fairway section between reporting points *KARSIBOR* and *DOK5* is considered. This section is 53.7 km (29 Nm) long. In the first half of 2009, in the north-south direction, 739 vessels were registered on this section. The registered vessel types and their numbers are listed in Table 1.

Table 1. The vessel number of different types on the *KARSIBOR* – *DOK5* section

Vessel type	Vessel number
Barges	27
Tankers	67
Containers	69
Cargo	366
General cargo	134
Carriers	49
Other vessels	27

Suppose that T is the random variable describing vessel traffic time on the *KARSIBOR – DOK5* section. Using formula (4), the probability density function for vessel passage speed on this section will be derived.

The vessel passage time as an extreme-valued random variable

In the work by Kasyk (Kasyk, 2016), the probability distribution of the passage time for general cargo vessels on the *KARSIBOR – DOK5* section was described. The hypothesis that the passage time for general cargo vessels is extreme-value distributed, was not rejected based on the Cramèr–von Mises goodness-of-fit test and chi-square goodness-of-fit test. Moreover, the location parameter $\alpha = 2.96$ and the scale parameter $\beta = 0.22$ are obtained.

From that and by formula (4), the probability density function of the passage speed $v(>0)$ for general cargo vessels is as follows:

$$g(v) = \frac{d}{v^2 \beta} \cdot \exp\left(\frac{\alpha}{\beta} - \frac{d}{v\beta} - \exp\left(\frac{\alpha}{\beta} - \frac{d}{v\beta}\right)\right) \quad (8)$$

where:

- d – fairway section length [Nm];
- α – location parameter of an extreme value distribution;
- β – scale parameter of an extreme value distribution.

The section between the reporting points *KARSIBOR* and *DOK5* is 53.7 km (29 Nm) long ($d = 29$ Nm). Thus, the probability density function of the passage speed for general cargo vessels is as follows:

$$g(v) = \frac{131.8}{v^2} \cdot \exp\left(13.45 - \frac{131.8}{v} - \exp\left(13.45 - \frac{131.8}{v}\right)\right) \quad (9)$$

One can calculate that the probability that the passage speed of general cargo vessels is less than 8.05 knots equals 0.05 and the probability that their

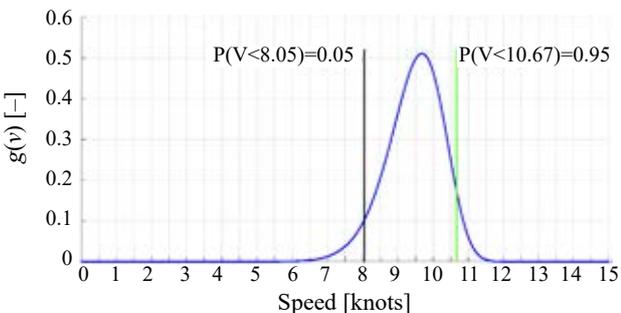


Figure 2. The probability density function $g(v)$ for general cargo vessel passage speed, where passage time is extreme-value distributed

passage speed is less than 10.67 knots equals 0.95. On the specified fairway section, the average value of the passage speed V for general cargo vessels equals 9.47 knots, with a standard deviation of 0.81 knots. The coefficient of variation, that is the standardized measure of dispersion, of the probability distribution for the passage speed of general cargo vessels equals 0.09. Therefore, one can conclude that the variation of the passage speed for general cargo vessels on the *KARSIBOR – DOK5* fairway is very little. It also confirms the skewness coefficient, which equals -0.49 . Both the value of this coefficient and the probability density function presented in Figure 2 indicate that the obtained random distribution of the passage speed V for general cargo vessels is left-skewed (slightly longer left tail). This means that the passage speed for most general cargo vessels is greater than the average speed of these vessels on the *KARSIBOR – DOK5* section. The kurtosis of this distribution equals 3.31 and its excess is equal to 0.31. This means that the obtained vessel passage speed distribution is leptokurtic.

The vessel passage time as a Weibull distributed random variable

In the work of Kasyk (Kasyk, 2016), it has been proved that the traffic time on the *KARSIBOR – DOK5* section for containers has a Weibull distribution with the following parameters: location parameter $\alpha = 1.6$, scale parameter $\beta = 0.33$ and shape parameter $\mu = 2.6$. From this, the container passage speed on this fairway section can be determined.

By formula (4), the probability density function of the container passage speed V , where the container passage time T is Weibull distributed, is as follows:

$$g(v) = \begin{cases} 0 & \text{for } v \leq 0 \\ \frac{\alpha d}{v^2} \left(\frac{1}{\beta}\right)^\alpha \left(\frac{d}{v} - \mu\right)^{-1+\alpha} \exp\left(-\left(\frac{1}{\beta}\right)^\alpha \left(\frac{d}{v} - \mu\right)^\alpha\right) & \text{for } v \in \left(0, \frac{d}{\mu}\right) \\ 0 & \text{for } v \geq \frac{d}{\mu} \end{cases} \quad (10)$$

that is,

$$g(v) = \frac{273.461}{v^2} \cdot \left(\frac{29}{v} - 2.6\right)^{0.6} \cdot \exp\left(-5.894 \left(\frac{29}{v} - 2.6\right)^{1.6}\right) \quad (11)$$

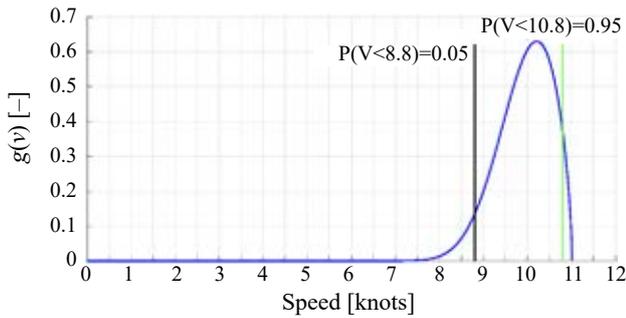


Figure 3. The probability density function $g(v)$ for the container passage speed, where their passage time is Weibull distributed

for $v \in (0, 11.15)$ and at $d = 29$ Nm, $\alpha = 1.6$, $\beta = 0.33$, $\mu = 2.6$. For other container passage speeds, the value of the probability density function g is equal to 0.

As illustrated in Figure 3, the presented random distribution of the passage speed V for containers has a negative coefficient of skewness (long left tail). This coefficient equals -0.58 . This means that the passage speed for most of the containers is greater than the average speed of these vessels on the *KARSIBOR – DOK5* section. On the *KARSIBOR – DOK5* section, the average value of the passage speed V for containers equals 9.93 knots. Moreover, the standard deviation equals 0.62 knots. The coefficient of variation of the probability distribution for the passage speed of containers equals 0.06. This value indicates that the variation of the passage speed for containers on the *KARSIBOR – DOK5* fairway section is very little. The kurtosis equals 2.89 and the excess is equal to -0.11 . This means that the obtained vessel passage speed distribution is platykurtic. The probability that the passage speed for containers is less than 8.8 knots equals 0.05 and the probability that their passage speed is less than 10.8 knots equals 0.95.

The vessel passage time as a Frèchet distributed random variable

In the last part of this section, the vessel passage time and speed for the group labeled “other vessels” from Table 1 is considered. Tugs, factory trawlers, research/survey vessels, suction dredgers, diving support vessels, ro-ro/passenger vessels, fishing vessels and offshore supply vessels make up this group. For this vessel group, data on their passage time from the semi-annual period were obtained from VTS Szczecin. Kasyk fitted the Frèchet distribution to such data. In the work of Kasyk (Kasyk, 2016), it has been proved that the traffic time on the *KARSIBOR – DOK5* section for the group labeled “other vessels” has the Frèchet distribution with the

following parameters: location parameter $\alpha = 7.59$, scale parameter $\beta = 1.84$, shape parameter $\mu = 1.09$. This was obtained using the most universal and popular statistical test: the Pearson chi-square goodness-of-fit test, and the stronger, but rarely used Cramèr–von Mises goodness-of-fit test. The author (Kasyk, 2016) pays attention to using a new distribution (specifically the Frèchet distribution) for the analysis of “other vessels” traffic flows to allow the building of better simulation models of these vessels’ traffic. Moreover, Kasyk (Kasyk, 2016) established that the difference between empirical and theoretical passage speeds for “other vessels” is less than 1%. In turn, the relative difference between empirical and theoretical variances of their passage speeds is less than 20%.

Based on Figure 8 (Frequency histogram of traffic time for other ships), presented in (Kasyk, 2016), it can be seen that the passage time distribution for “other vessels” is not symmetrical (as it would be in the case of the Normal distribution), but it is right-skewed (extended right tail). According to this, and using formula (3), it is expected that the passage speed distribution for “other vessels” is also not symmetrical. Moreover, it is expected that this distribution is left-skewed (extended left tail), since the passage time and passage speed are inversely proportional.

By formula (4), the probability density function of the “other vessels” passage speed V , where the “other vessels” passage time T is Frèchet distributed, is as follows:

$$g(v) = \frac{\alpha d}{v^2} \cdot \left(\frac{1}{\beta}\right)^{-\alpha} \left(\frac{d}{v} - \mu\right)^{-1-\alpha} \cdot \exp\left[-\left(\frac{1}{\beta}\right)^{-\alpha} \left(\frac{d}{v} - \mu\right)^{-\alpha}\right] \quad (12)$$

for $v > 0$, that is,

$$g(v) = \frac{22522}{v^2 \left(\frac{29}{v} - 1.09\right)^{8.59}} \cdot \exp\left[-\frac{102.322}{\left(\frac{29}{v} - 1.09\right)^{7.59}}\right] \quad (13)$$

for $d = 29$ Nm, $\alpha = 7.59$, $\beta = 1.84$ and $\mu = 1.09$.

In Figure 4, the plotted random distribution of the passage speed V for the group labeled “other vessels” has also a negative coefficient of skewness (long left tail). The skewness coefficient is equal to -0.88 . Thus, the passage speed for most of the vessels from this group is greater than the average speed

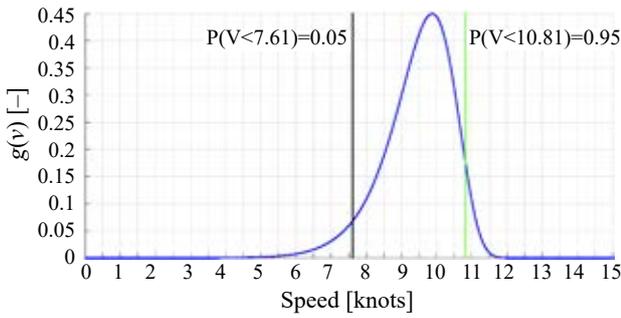


Figure 4. The probability density function $g(v)$ for the passage speed of the “other vessels” group, where their passage time is Frèchet distributed

of these vessels on the *KARSIBOR – DOK5* section. The authors think that such asymmetry results from the speed limits introduced on the Szczecin–Świnoujście fairway.

On this section, the average value of the passage speed V for this vessel group equals 9.46 knots and its standard deviation equals 0.99 knots. The coefficient of variation of the probability distribution for the passage speed of this vessel group equals 0.1. This value indicates that the variation of the passage speed for this vessel group on the *KARSIBOR – DOK5* fairway is small, but it is greater than the variation of the passage speed for general cargo vessels and containers. The kurtosis is equal to 4.22 and the excess equals 1.22. This means that the obtained vessel passage speed distribution is leptokurtic. The probability that the passage speed for this vessel group is less than 7.61 knots equals 0.05 and the probability that their passage speed is less than 10.81 knots equals 0.95.

Results and discussion

The determined probability density functions allow estimation of vessel speeds along with the probability of specific speeds occurring. For example, from the aspect of navigation safety, one can

determine the probability that some vessels exceed the allowed average speed on the *KARSIBOR – DOK5* fairway section. The *KARSIBOR – DOK5* fairway section consists of eight sections with certain speed limits. Some of them are limited to 8 knots, and others to 12 knots.

Table 2. *KARSIBOR – DOK5* section parameters and vessel speeds on this section

Fairway section	Section length [km]	Allowed speed [knots]
KARSIBOR – 11 KM	2.1	8
11 KM – I BT	5.2	8
I BT – CHEŁMINEK	19.2	12
CHEŁMINEK – PŁAWY	1.5	8
PŁAWY – KRĘPA DOLNA	8.3	12
KREPA DOLNA – RADUŃ GÓRNA	3.0	8
RADUŃ GÓRNA – INOUJŚCIE	6.0	12
INOJŚCIE – DOK5	8.4	8

The total sum of sections with the allowed speed of 12 knots is 33.6 km, and the total sum of sections with the allowed speed of 8 knots is 20.1 km. Thus, the average speed allowed for the entire section to be analyzed should not exceed 10.1. The probability of exceeding this speed can be determined on the basis of the obtained probability density functions for the vessel passage speed. Thus, in individual cases, it is as follows:

- for general cargo vessels (14);
- for containers (15);
- for “other vessels” (16).

The assigned probabilities indicate that a significant percentage of vessels exceed the designated speed limits. This is particularly glaring in the case of containers, where over 50% of these vessels do not conform to the regulations. A more detailed analysis of the obtained distributions allows us to evaluate the size of these exceedances. Thus, 6.2% of

$$\int_{10.1}^{\infty} g(v)dv = \int_{10.1}^{\infty} \frac{131.8}{v^2} \cdot \exp\left(13.45 - \frac{131.8}{v} - \exp\left(13.45 - \frac{131.8}{v}\right)\right) dv = 0.224 \quad (14)$$

$$\int_{10.1}^{11.1538} g(v)dv = \int_{10.1}^{11.1538} \frac{273.461}{v^2} \cdot \left(\frac{29}{v} - 2.6\right)^{0.6} \cdot \exp\left(-5.894\left(\frac{29}{v} - 2.6\right)^{1.6}\right) dv = 0.519 \quad (15)$$

$$\int_{10.1}^{\infty} g(v)dv = \int_{10.1}^{\infty} \frac{22522}{v^2 \left(\frac{29}{v} - 1.09\right)^{8.59}} \cdot \exp\left(-\frac{102.322}{\left(\frac{29}{v} - 1.09\right)^{7.59}}\right) dv = 0.278 \quad (16)$$

general cargo vessels, 21.2% of containers and 9.6% of “other vessels” exceed the speed limit by 5% of the permissible value, i.e., exceeding the speed of 10.605 knots. 0.7% of general cargo vessels, 0.4% of containers and 1.4% of “other vessels” exceed the speed limit by 10% of the allowed value, i.e., exceeding the speed of 11.11 knots. With appropriately estimated losses, the above-mentioned values are necessary elements for estimating the navigational risk.

Conclusions

In this paper, the authors have presented a certain approach for modeling vessel passage speeds in selected restricted areas. Vessel passage speed is modeled as the inverse random variable. Knowing the probabilistic model for vessel passage time, the probabilistic model for vessel passage speed can be calculated. This method could be used to analyze vessel traffic flows. Describing the vessel traffic flow is crucial for maintaining navigation safety, managing marine transportation, and performing computer simulations using the simulation models that have been developed.

The obtained probabilistic models for vessel passage speed extend the possibility of describing vessel speeds on the chosen waterway section, where only the vessel passage time is known. Sometimes, this situation occurs when the vessel report is registered only at selected points of the fairway.

The obtained probabilistic models for the vessel passage speed enable the calculation of the likelihoods for different random events, e.g., exceeding or not exceeding the permitted speed on the selected waterway sections. This is especially important when the waterway is subject to disruptions such as speed limits and other restrictions.

In situations when no disruptions occur or a vessel traffic flow is not too much disturbed, the vessel passage time is modeled using a normal distribution; in this case, the vessel passage speed distribution has positive coefficients of skewness (longer right tail). In the three studied cases, concerning vessel traffic on the Szczecin–Świnoujście fairway, the random distributions of the passage speed for the chosen vessel groups have negative skewness (longer left tail). On this fairway, there is a speed limit: vessels can pass with a maximum speed of 8 or 12 knots. On the basis of the assigned probabilities, one can observe that a significant percentage of vessels exceeded the designed speed limits on the specified Szczecin–Świnoujście fairway sections.

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